Leaving Cert Physics Long Questions 2018 – 2002

4. Circular motion and SHM *(all higher level)*

Remember to photocopy 4 pages onto 1 sheet by going A3→A4 and using back to back on the photocopier

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Circular motion

2012 Question 12 (a)

An Olympic hammer thrower swings a mass of 7.26 kg at the end of a light inextensible wire in a circular motion. In the final complete swing, the hammer moves at a constant speed and takes 0.8 s to complete a circle of radius 2.0 m.

(i) What is the angular velocity of the hammer during its final swing?
(ii) Even though the hammer moves at a constant speed, it accelerates. Explain.
(iii) Calculate the acceleration of the hammer during its final swing
(iv) Calculate the kinetic energy of the hammer as it is released.

2018 Question 6 (a)

During the discus event, Ashton swings a discus of mass 2.0 kg in uniform circular motion. The radius of orbit of the discus is 1.2 m and the discus has a velocity of 20.4 m s^{-1} when Ashton releases it.

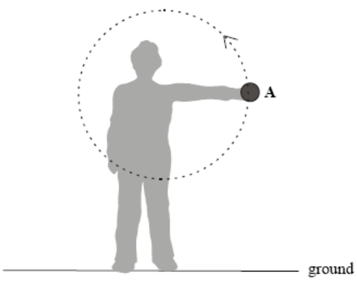
(i) Derive an expression to show the relationship between the radius, velocity and angular velocity of an object moving in uniform circular motion.
(ii) Calculate the angular velocity of the discus immediately prior to its release.
(iii) Calculate the centripetal force acting on the discus just before Ashton releases it.
(iv) In what direction does this force apply?

2011 Question 6 (*c*)

A simple merry-go-round consists of a flat disc that is rotated horizontally.
A child of mass 32 kg stands at the edge of the merry-go-round, 2.2 metres from its centre.
The force of friction acting on the child is 50 N.
Draw a diagram showing the forces acting on the child as the merry-go-round rotates.

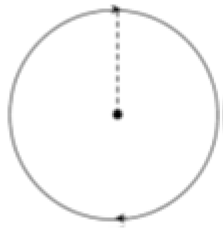
What is the maximum angular velocity of the merry-go-round so that the child will not fall from it, as it rotates?
If there was no force of friction between the child and the merry-go-round, in what direction would the child move as the merry-go-round starts to rotate?

2006 Question 6



(i) Define velocity.
(ii) Define angular velocity.
(iii) Derive the relationship between the velocity of a particle travelling in uniform circular motion and its angular velocity.
(iv) A student swings a ball in a circle of radius 70 cm in the vertical plane as shown. The angular velocity of the ball is 10 rad s^{-1}.
What is the velocity of the ball?
(v) How long does the ball take to complete one revolution?
(vi) Draw a diagram to show the forces acting on the ball when it is at position A.
(vii) The student releases the ball when is it at A, which is 130 cm above the ground, and the ball travels vertically upwards. Calculate the maximum height, above the ground, the ball will reach.
(viii) Calculate the time taken for the ball to hit the ground after its release from A.

2016 Question 12 (c)
{The other part of 12 (c) was related to the Doppler effect, which I put into the "Waves, Sound and Light" long questions}



(i) Define centripetal force.
(ii) A buzzer attached to a string of length 80 cm moves at a speed of 13 m s^{-1} in a vertical circle.

The buzzer has a mass of 70 g. Calculate the maximum and minimum tension in the string.
{This is another sh1t question, because the speed at the bottom should be greater than the speed at the top, but here they simply tell you that it is 13 m s^{-1} in both cases.}

Circular motion and gravity

2004 Question 12 (a)

(i) State Newton's universal law of gravitation.

(ii) Centripetal force is required to keep the earth moving around the sun.
What provides this centripetal force?

(iii) In what direction does this centripetal force act?

(iv) Give an expression for centripetal force.

(v) The earth has a speed of 3.0×10^4 m s^{-1} as it orbits the sun.
The distance between the earth and the sun is 1.5×10^{11} m
Calculate the mass of the sun.
(gravitational constant $G = 6.7 \times 10^{-11}$ m^3 kg^{-1} s^{-2})

2013 Question 6

(i) State Newton's law of universal gravitation.

(ii) Explain what is meant by angular velocity.

(iii) Derive an equation for the angular velocity of an object in terms of its linear velocity when the object moves in a circle.

The International Space Station (ISS), shown in the photograph, functions as a research laboratory and a location for testing of equipment required for trips to the moon and to Mars.
The ISS orbits the earth at an altitude of 4.13×10^5 m every 92 minutes 50 seconds.

(iv) Calculate (*a*) the angular velocity, (*b*) the linear velocity, of the ISS.

(v) Name the type of acceleration that the ISS experiences as it travels in a circular orbit around the earth.

(vi) What force provides this acceleration?

(vii) Calculate the attractive force between the earth and the ISS.

(viii) Hence or otherwise, calculate the mass of the earth.

(ix) If the value of the acceleration due to gravity on the ISS is 8.63 m s^{-2}, why do occupants of the ISS experience apparent weightlessness?

(x) A geostationary communications satellite orbits the earth at a much higher altitude than the ISS.
What is the period of a geostationary communications satellite?
(mass of ISS = 4.5×10^5 kg; radius of the earth = 6.37×10^6 m)

2008 Question 6

(i) State Newton's law of universal gravitation.

(ii) The international space station (ISS) moves in a circular orbit around the equator at a height of 400 km.
What type of force is required to keep the ISS in orbit?

(iii) What is the direction of this force?

(iv) Calculate the acceleration due to gravity at a point 400 km above the surface of the earth.

(v) An astronaut in the ISS appears weightless. Explain why.

(vi) Derive the relationship between the period of the ISS, the radius of its orbit and the mass of the earth.

(vii) Calculate the period of an orbit of the ISS.

(viii) After an orbit, the ISS will be above a different point on the earth's surface. Explain why.

(ix) How many times does an astronaut on the ISS see the sun rise in a 24 hour period?
(gravitational constant = 6.6×10^{-11} N m^2 kg$^{-2;}$ mass of the earth = 6.0×10^{24} kg;
radius of the earth = 6.4×10^6 m)

2005 Question 6

(i) Define angular velocity.

(ii) Define centripetal force.

(iii) State Newton's Universal Law of Gravitation.

(iv) A satellite is in a circular orbit around the planet Saturn.
Derive the relationship between the period of the satellite, the mass of Saturn and the radius of the orbit.

(v) The period of the satellite is 380 hours. Calculate the radius of the satellite's orbit around Saturn.

(vi) The satellite transmits radio signals to earth. At a particular time the satellite is 1.2×10^{12} m from earth.
How long does it take the signal to travel to earth?

(vii) It is noticed that the frequency of the received radio signal changes as the satellite orbits Saturn.
Explain why.

Gravitational constant = 6.7×10^{-11} N m^2 kg^{-2}
mass of Saturn = 5.7×10^{26} kg
speed of light = 3.0×10^8 m s^{-1}

2015 Question 6

In the circular orbit of a satellite around the Earth, the required centripetal force is the gravitational force between the satellite and the Earth.
The force can be determined using Newton's law of universal gravitation.

(i) Explain what is meant by centripetal force.

(ii) State Newton's law of universal gravitation.

(iii) Derive the relationship between the period of a satellite, the radius of its orbit and the mass of the Earth.

A Global Positioning Systems (GPS) receiver can calculate its position on Earth to within a few metres.
It picks up radio-wave signals from several of the 32 GPS satellites orbiting the Earth.
GPS satellites orbit the Earth in Medium Earth Orbit (MEO) with a period of 12 hours.

(iv) Calculate the height of a GPS satellite above the Earth's surface.

(v) Calculate the speed of a GPS satellite.

(vi) Calculate the minimum time it takes a GPS signal to travel from the satellite to a receiver on the surface of the Earth.

(vii) Explain why GPS satellites are not classed as geostationary satellites.

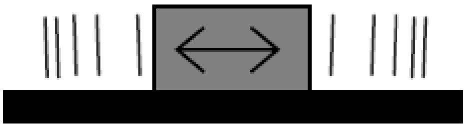
(viii) Radio-waves, such as those used by GPS satellites, have the lowest frequency of all electromagnetic radiation types.
What type of electromagnetic radiation has the next lowest frequency?

(mass of Earth = 5.97×10^{24} kg; radius of Earth = 6371 km)

Simple harmonic motion

2011 Question 12 (a)

State Hooke's law.



A body of mass 250 g vibrates on a horizontal surface and its motion is described by the equation $a = -16\,s$, where s is the displacement of the body from its equilibrium position.

The amplitude of each vibration is 5 cm.

(a) Why does the body vibrate with simple harmonic motion?

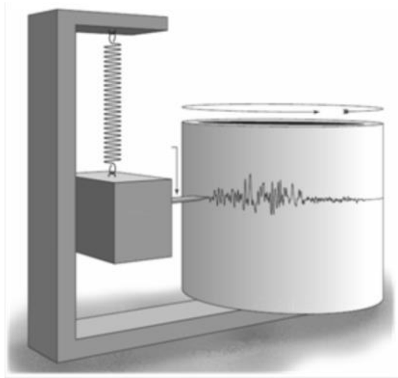
(b) Calculate the frequency of vibration of the body?

(c) What is the magnitude of (i) the maximum force, (ii) the minimum force, which causes the body's motion?

2013 Question 11

Read the following passage and answer the accompanying questions.

A seismometer consists of a sensor that detects ground motion, attached to a recording system.
A seismometer that is sensitive to up-down motions of the ground, as caused by an earthquake, can be understood by visualising a mass hanging on a spring as shown in the diagram.
The frame and the drum move up and down as the seismic wave passes by, but the mass remains stationary.
If a recording system is installed, such as a rotating drum attached to the frame and a pen attached to the mass, this relative motion between the suspended mass and the ground can be recorded to produce a seismogram, as shown in the diagram.



Modern seismometers do not use a pen and drum.
The relative motion between a magnet that is attached to the mass, and the frame, generates a potential difference that is recorded by a computer.
(Adapted from *www.iris.edu Education and Outreach Series No.7: How does a Seismometer Work?*)

(i) Seismic waves can be longitudinal or transverse.
What is the main difference between them?

(ii) An earthquake generates a seismic wave that takes 27 seconds to reach a recording station. If the wave travels at 5 km s^{-1} along the earth's surface, how far is the station from the centre of the earthquake?

(iii) Draw a diagram to show the forces acting on the suspended mass when the seismometer is at rest.

(iv) At rest, the tension in the spring is 49 N. What is the value, in kilograms, of the suspended mass?

(v) What type of motion does the frame have when it moves relative to the mass?

(vi) During an earthquake the ground was observed at the recording station to move up and down as the seismic wave generated by the earthquake passed. Give an equation for the acceleration of the ground in terms of the periodic time of the wave motion and the displacement of the ground.

(vii) If the period of the ground motion was recorded as 17 seconds and its amplitude was recorded as 0.8 cm, calculate the maximum ground acceleration at the recording station.

(viii) In some modern seismometers a magnet is attached to the mass and a coil of wire is attached to the frame. During an earthquake, there is relative motion between the magnet and the coil.
Explain why an emf is generated in the coil.

(ix) (acceleration due to gravity, $g = 9.8$ m s^{-2})

Simple harmonic motion and Hooke's law

Tricky maths questions - points to note

- When using the $F = k\,s$ expression for Hooke's law, note that s represents the extension, i.e. the distance between the *new length* and *the end of the natural length*.
  However when using the expression for simple harmonic motion ($a = -\omega^2 s$) *s represents the distance between the new length and the equilibrium position.*
- Remember that the most common equation used here is the following: $\boxed{\omega = \sqrt{\frac{k}{m}}}$

2017 Question 6

In a bungee jump, Henry falls while attached to an elastic cord.
When the cord stops Henry's fall, he then oscillates up and down.
During the bungee jump, gravitational potential energy is converted into kinetic energy and then into elastic potential energy.

(i) State the principle of conservation of energy.
(ii) Derive the expression $v^2=u^2+2as$ for uniform accelerated motion.
(iii) The cord is 32 m long and Henry, of mass 60 kg, falls from rest while attached.
Calculate his speed when he has fallen 16 m.

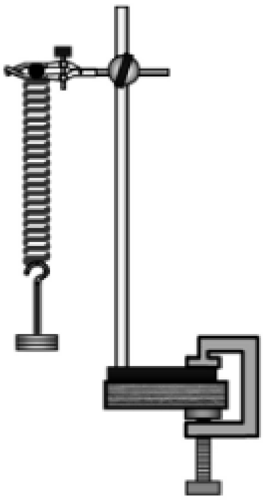
A stretched elastic cord obeys Hooke's law and the weight attached to the cord oscillates with simple harmonic motion.

(iv) State Hooke's law.
(v) What is meant by simple harmonic motion?
(vi) The elastic constant of the cord is 250 N m^{-1}.
Calculate the length the cord would have if Henry was suspended at rest.

After the end of the fall, Henry oscillates with simple harmonic motion.
The maximum displacement from his rest position is 1.2 metres.

(vii) Calculate (*i*) his maximum acceleration as he oscillates and (*ii*) his period of oscillation.
(viii) Draw a diagram to show the forces acting on Henry when he is at his lowest point.
(*acceleration due to gravity, $g = 9.8$ m s^{-2}*)

2014 Question 12 (a)



(i) State Hooke's law.
(ii) The elastic constant of a spring is 12 N m^{-1} and it has a length of 25 mm.
An object of mass 20 g is attached to the spring.
What is the new length of the spring?
(iii) The object is then pulled down until the spring's length is increased by a further 5 mm and is then released. The object oscillates with simple harmonic motion.
Sketch a velocity-time graph of the motion of the object.
(iv) Calculate the period of oscillation of the object.
(acceleration due to gravity, $g = 9.8$ m s^{-2})

2009 Question 12 (a)

(i) State Hooke's law.

(ii) When a sphere of mass 500 g is attached to a spring of length 300 mm, the length of the spring increases to 330 mm.
Calculate the spring constant.

(iii) The sphere is then pulled down until the spring's length has increased to 350 mm and is then released.
Describe the motion of the sphere when it is released.

(iv) What is the maximum acceleration of the sphere?
(acceleration due to gravity = 9.8 m s^{-2})

2007 Question 6

(i) State Hooke's law.

(ii) A stretched spring obeys Hooke's law.
When a small sphere of mass 300 g is attached to a spring of length 200 mm, its length increases to 285 mm.
Calculate its spring constant.

(iii) The sphere is pulled down until the length of the spring is 310 mm.
The sphere is then released and oscillates about a fixed point.
Derive the relationship between the acceleration of the sphere and its displacement from the fixed point.

(iv) Why does the sphere oscillate with simple harmonic motion?

(v) Calculate the period of oscillation of the sphere

(vi) Calculate the maximum acceleration of the sphere

(vii) Calculate the length of the spring when the acceleration of the sphere is zero.
(acceleration due to gravity = 9.8 m s^{-2})

2018 Question 12 (a)

A simple pendulum can execute <u>simple harmonic motion</u>.

(i) Explain the underlined term.

(ii) When does a simple pendulum execute simple harmonic motion?

(iii) What is the relationship between the period and the length of a simple pendulum?

A stretched spring can also execute simple harmonic motion.
A spring has a natural length of 50 cm.
A mass of 60 g is hung from the spring and the mass is allowed to oscillate with simple harmonic motion.
It has a period of oscillation of 0.85 seconds.

(iv) Calculate the spring constant,

(v) Calculate the length of the spring when the mass is at rest.
(acceleration due to gravity = 9.8 m s^{-2})

2002 Question 6

(i) State Newton's second law of motion.

(ii) The equation F = – ks, where k is a constant, is an expression for a law that governs the motion of a body.
Name this law and give a statement of it.

(iii) Give the name for this type of motion and describe the motion.

(iv) A mass at the end of a spring is an example of a system that obeys this law.
Give two other examples of systems that obey this law.

(v) The springs of a mountain bike are compressed vertically by 5 mm when a cyclist of mass 60 kg sits on it.
When the cyclist rides the bike over a bump on a track, the frame of the bike and the cyclist oscillate up and down.
Using the formula F = – ks, calculate the value of k, the constant for the springs of the bike.

(vi) The total mass of the frame of the bike and the cyclist is 80 kg.
Calculate (i) the period of oscillation of the cyclist, (ii) the number of oscillations of the cyclist per second. (acceleration due to gravity, g = 9.8 m s^{-2})

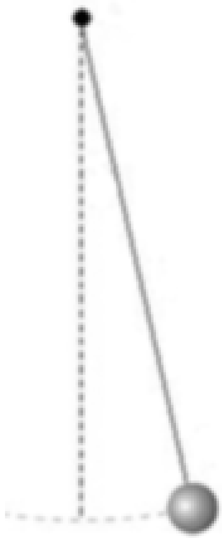
2016 Question 6

A mass at the end of a spring obeys Hooke's law.
The mass can be made to oscillate vertically, so that it executes <u>simple harmonic motion</u>.

(i) Explain the underlined term.

(ii) State Hooke's law.

(iii) Use Hooke's law to show that the mass executes simple harmonic motion.



A simple pendulum also executes simple harmonic motion.
The time taken for each oscillation of a certain simple pendulum on the Earth's surface is 2 s.
The weight of its bob is 3.5 N.
The bob travels along a curved path.
It travels a distance of 18 cm during each oscillation.

(iv) Calculate the length of the pendulum

(v) Calculate the maximum angular displacement of the pendulum.

(vi) Draw a diagram to show the forces acting on the bob when it is at its maximum displacement.

(vii) Calculate the restoring force at this point.

(viii) At what point during its movement does the bob have its greatest angular velocity?

(ix) The period of a simple pendulum varies with its height above the surface of the Earth.
At what height will the period of a simple pendulum be 2% more than the period of a simple pendulum of the same length at the Earth's surface?
Show your work clearly.

(acceleration due to gravity at the Earth's surface = 9.8 m s^{-2}; radius of Earth = 6371 km)

Solutions

2018 Question 12 (a)

(i) **Explain the term *simple harmonic motion.***
An object is undergoing simple harmonic motion if its acceleration towards a fixed point is proportional to its displacement.

(ii) **When does a simple pendulum execute simple harmonic motion?**
When it is oscillating at a small angle.

(iii) **What is the relationship between the period and the length of a simple pendulum?**
The period squared is proportional to the length.

(iv) **Calculate the spring constant**

$T = \frac{2\pi}{\omega}$ $\omega = \frac{2\pi}{T}$ $= \frac{2\pi}{0.85}$ = 7.39

$\omega^2 = \frac{k}{m}$ $k = \omega^2 m$ $= (7.39)^2(0.06) = 3.28$ N m^{-1}

(v) **Calculate the length of the spring when the mass is at rest.**
F = -k (extension)
mg = -k (extension) {we can ignore the minus sign – it exists merely to signify that the restoring force and the extension are opposite in direction}
(0.06)(9.8) = 3.28(extension)
Extension = 0.18 m
Length of string = 0.5 + 0.18 = 0.68 m

2018 Question 6 (a)

(i) **Derive an expression to show the relationship between the radius, velocity and angular velocity of an object moving in uniform circular motion.**

$\theta_{(in\ radians)} = \frac{arc\ length}{radius}$ $\Rightarrow$ $\theta = \frac{s}{r}$

$\frac{\theta}{t} = \frac{s}{tr}$ {divided both sides by t}

$\frac{\theta}{t} = \frac{s}{t} \times \frac{1}{r}$ but $\frac{\theta}{t} = \omega$ and $\frac{s}{t} = v$

$\omega = v \times \frac{1}{r}$ $\Rightarrow$ $v = r\omega$

(ii) **Calculate the angular velocity of the discus immediately prior to its release.**

$\omega = \frac{v}{r} = \frac{20.4}{1.2}$ = 17 rads s^{-1}

(iii) **Calculate the centripetal force acting on the discus just before Ashton releases it.**
$F = mr\omega^2 = (2)(1.2)(17)^2 = 693.6$ N

(iv) **In what direction does this force apply?**
Towards the centre

2017 Question 6

(i) State the principle of conservation of energy.

Energy cannot be created or destroyed but only converted from one form to another

(ii) Derive the expression $v^2=u^2+2as$ for uniform accelerated motion.

$v = u + at$ $\Rightarrow$ {multiply out both sides} $(v)^2 = (u + at)^2$ $\Rightarrow$ $v^2 = u^2 + 2uat + (at)^2$

$v^2 = u^2 + 2a(ut + ½ at^2)$

$v^2 = u^2 + 2as$ {because $s = ut + ½ at^2$}

(iii) Calculate his speed when he has fallen 16 m.

$v^2 = u^2 + 2as$ $v^2 = 0 + 2(9.8)(16)$ $v = 17.7$ m s^{-1}

(iv) State Hooke's law

Hooke's law states that when a force is exerted to extend or compress a spring, the *restoring force* is proportional to the displacement.

(v) What is meant by simple harmonic motion?

An object is said to be moving with Simple Harmonic Motion if its acceleration is directly proportional to its displacement *from* a fixed point in its path, and its acceleration is directed *towards* that point.

(vi) Calculate the length the cord would have if Henry was suspended at rest.

If Henry is at rest then the force upwards must equal the force downwards.

The force upwards is the tension in the string, which we know from Hooke's is proportional to the extension (d), so $F_{upwards} = kd$, where k is the proportional constant. In this case k corresponds to the elastic constant of the cord (250 N m^{-1}).

$F_{upwards} = (250)d$

The force downwards is simply his weight: $mg = (9.8)(60)$

$(250)d = (9.8)(60)$

$d = 2.352$ m = extension.

So the overall length of the cord = 32 + 2.35 = 34.35 m

(vii) Calculate (*i*) his maximum acceleration as he oscillates and (*ii*) his period of oscillation.

Maximum acceleration: From the equation $a = -\omega^2 x$ the acceleration will be a maximum when the displacement (x) is a maximum. The question tells us that this maximum displacement is 1.2 metres.

Note that for simple harmonic motion we can use the relationship $\boldsymbol{\omega}^2 = \frac{k}{m}$

$\boldsymbol{\omega}^2 = \frac{k}{m} = 250/60 = 4.17$ $a = -\omega^2 x$ $a = -(4.17)(1.2)$

$a = -5$ m s^{-2}

His period of oscillation:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{4.17}} = 3.08\ s$$

(viii) Draw a diagram to show the forces acting on Henry when he is at his lowest point.

Arrow showing force down

Arrow showing force up

It must be clearly indicated that the force upwards is greater than the force downward

2016 Question 12 (c)

(i) What is meant by the Doppler effect?

Apparent change in frequency of a wave due to relative motion between source and observer

(ii) Define centripetal force.

Centripetal force is the force acting towards the centre on an object moving in a circle.

(iii) Calculate the maximum and minimum frequency of the note detected by an observer

u = 13 m s^{-1}

f = 1.1 kHz = 1100 Hz

The frequency of the note detected by an observer is a *maximum* as the buzzer moves *away from* the observer, so we use the positive sign below the line.

$$f' = \frac{fc}{c+u}$$

$$f' = \frac{(1100)(340)}{340+13}$$

f'_{max} = 1143.7 Hz

The frequency of the note detected by an observer is a *minimum* as the buzzer moves *towards* the observer, so we use the negative sign below the line.

$$f' = \frac{fc}{c-u}$$

$$f' = \frac{(1100)(340)}{340-13}$$

f'_{min} = 1059.5 Hz

(iv) Calculate the maximum and minimum tension in the string.

m = 70 grams = 0.07 kg
r = 80 cm = 0.8 m
v = 13 m s^{-1}

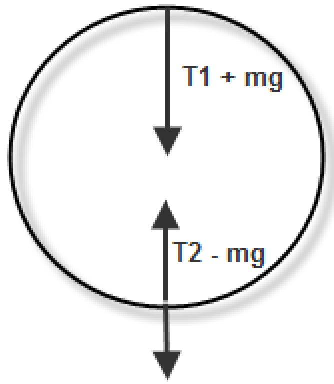
{Here we use our basic equation for circular motion:

$$F_{net} = \frac{mv^2}{r}$$

Note that there are two forces acting on the mass when it is in both positions; tension and weight (mg)
The tension will be different in both cases.

At the top, both forces are acting downwards, so we add them together to get $T_1 + mg = \frac{mv^2}{r}$

At the bottom, tension is acting upwards and weight is acting downwards.
The tension is the bigger force because this is what is what is responsible for the centripetal motion, so we use $T_2 - mg = \frac{mv^2}{r}$*}*



At the top:

$$T_1 + mg = \frac{mv^2}{r} \qquad T_1 = \frac{mv^2}{r} - mg$$

$$T_1 = \frac{(0.07)13^2}{0.8} - (0.07)(9.8)$$

T_{min} = 14.1 N

At the bottom:

$$T_2 - mg = \frac{mv^2}{r} \qquad T_2 = \frac{mv^2}{r} + mg$$

$$T_2 = \frac{(0.07)13^2}{0.8} + (0.07)(9.8)$$

T_{max} = 15.5 N

{Just to reinforce the point that the velocities should be different at the top and bottom, note that a similar question appeared on the higher level Applied Maths paper in 1995}

A particle of mass m, attached to a fixed point by a light inelastic string, describes a circle in a vertical plane.

The tension of the string when the particle is at the highest point of the orbit is T_1 and when at the lowest point it is T_2.

Prove that $T_2 = T_1 + 6mg$

Solution

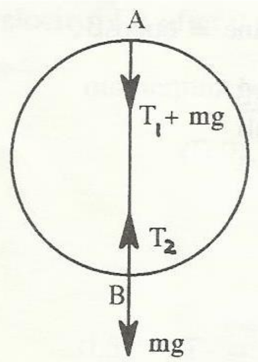
See diagram.

Note that they could have used

$$F = \frac{mv^2}{r} \qquad or \qquad F = mr\omega^2$$

In this case they used $F = mr\omega^2$

Both options would have given the same result.



$$T_1 + mg = mr\omega_1^2$$

$$T_2 - mg = mr\omega_2^2$$

Energy at A = Energy at B

$$0.5mr^2\omega_1^2 + mg(2r) = 0.5mr^2\omega_2^2$$

$$0.5(T_1 + mg) + 2mg = 0.5(T_2 - mg)$$

$$\Rightarrow T_2 = T_1 + 6mg$$

2016 Question 6

(i) Explain the underlined term.

An object exhibits simple harmonic motion if its acceleration proportional to its displacement and is opposite in direction.

(ii) State Hooke's law.

For a stretched string, the restoring force is proportional to displacement, and restoring force and displacement are opposite in direction.

(iii) Use Hooke's law to show that the mass executes simple harmonic motion.

$F = -ks$

$ma = -ks$

$a = -\frac{k}{m}s$

This is consistent with simple harmonic motion

(iv) Calculate the length of the pendulum

$T^2 = \frac{4\pi^2 l}{g}$ $l = \frac{gT^2}{4\pi^2}$ $l = \frac{(9.8)2^2}{4\pi^2}$ $l = 0.99$ m

(v) Calculate the maximum angular displacement of the pendulum.

{"Calculate the maximum angular displacement" means "calculate the largest angle that the pendulum will be from the equilibrium".

Equilibrium position is where the pendulum would be when it stops - in this case the pendulum would come to rest at the bottom of its cycle. So maximum angular displacement is the angle between this point and the top of the pendulum's cycle.

We are told that the pendulum travels a distance of 18 cm during each oscillation. One oscillation corresponds to one full cycle – all the way over and all the way back. The arc length between equilibrium position and the top of the pendulum's cycle therefore corresponds to one quarter of the full cycle}

$\theta = \frac{arc\ length}{radius}$ arc length = (¼)(0.18) = 0.045 m

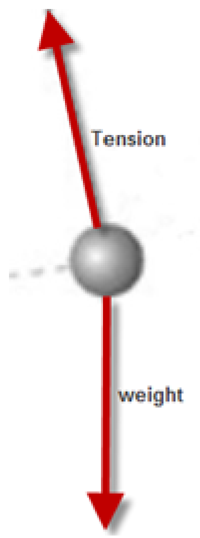
and the radius corresponds to the length of the pendulum: 0.99 m

$\theta = \frac{0.045}{0.99}$ = 0.045 radians

(vi) Draw a diagram to show the forces acting on the bob when it is at its maximum displacement.

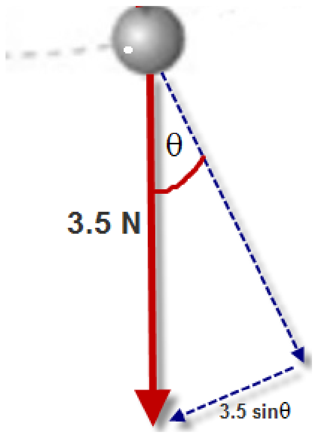
Weight down

Tension up at an angle to the vertical



(vii) Calculate the restoring force at this point.

{The restoring force is the force acting in towards equilibrium position.
There doesn't seem to be any force acting in this direction, but there is a component of the weight acting inwards.
To find out the size of this inward component we resolve the 3.5 N into 2 components - one acting in the direction we're interested in and the other at right angles.
From the diagram we can see that the component acting inward corresponds to 3.5 sin θ}



F = 3.5 sin 0.045 {remember to set your calculator to 'radians' mode, because our answer for θ was in radians}

F = 0.16 N

(viii) At what point during its movement does the bob have its greatest angular velocity?

When θ = 0 / at the centre of oscillation / at its lowest point

(ix) At what height will the period of a simple pendulum be 2% more than the period of a simple pendulum of the same length at the Earth's surface?

Where do I even begin with this bast@rd of a question?

$T = 2\pi\sqrt{\frac{l}{g}}$ $\quad T^2 = 4\pi^2\frac{l}{g}$ $\quad g = 4\pi^2\frac{l}{T^2}$ $\quad g \propto \frac{1}{T^2}$

Now from our chapter on *Gravity* we had the following relationship: $g = \frac{Gm}{d^2}$ $\quad g \propto \frac{1}{d^2}$

So if $g \propto \frac{1}{T^2}$ and $g \propto \frac{1}{d^2}$

then {*and I'm not saying this is obvious, or that anything like it was on any past paper or even on the syllabus*} mathematically we can say that $T \propto d$

So if we want T to increase by 2% more than it would be on the surface of the Earth, then we need d to increase by 2%.
(d represents the radius of the Earth = 6371 km)

2% of 6371000 = 127.4 km

So new height = radius of the Earth + 127.4 km (i.e. 127.4 km above the surface of the Earth).

2015 Question 6

(i) Explain what is meant by centripetal force.
The force - acting in towards the centre - required to keep an object moving in a circle is called centripetal force.

(ii) State Newton's law of universal gravitation.
Newton's law of gravitation states that any two point masses in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

(iii) Derive the relationship between the period of a satellite, the radius of its orbit and the mass of the Earth.

Centripetal force = gravitational force

$$\frac{\mathrm{m}v^2}{r} = \frac{\mathrm{G}m_1 m_2}{d^2}$$

Cancel one *m* on both sides. The *r* and *d* both correspond to the same thing, so we can also cancel the *r* on the left hand side with one of the *d*'s on the right hand side.

So we are left with $v^2 = \frac{Gm}{r}$ *Equation (1)*

$$speed = \frac{distance}{time}$$

Distance in this case is the circumference of a circle (2πR for circular satellite orbits)
The time corresponds to the time for one compete orbit, so we give it the symbol *T*.
We also allow *v* to represent speed (even though technically *v* represents velocity, which is not the same)

$\Rightarrow$ $v = \frac{2\pi R}{T}$ $\Rightarrow$ (square both sides) $v^2 = \frac{4\pi^2 R^2}{T^2}$ *Equation (2)*

Equating Equations (1) and (2) we get

$$\frac{\mathrm{Gm}}{\mathrm{r}} = \frac{4\pi^2 R^2}{T^2}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 R^3}{GM}$$

(iv) Calculate the height of a GPS satellite above the Earth's surface.

T = 12 hours = (12)(60)(60) = 43200 seconds
Mass of Earth = 5.97×10^{24} kg
Radius of Earth = 6.371×10^{6} m
G = 6.7×10^{-11} N m^2 kg^{-2}

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$R^3 = \frac{GMT^2}{4\pi^2}$$

$$R^3 = \frac{(6.7\times10^{-11})(5.97\times10^{24})(43200)^2}{4\pi^2}$$

R = 2.66×10^{7} m
h = (2.66×10^{7} – radius of the earth) = 2.023×10^{7} m

(v) Calculate the speed of a GPS satellite.

$$v^2 = \frac{GM}{R}$$

$$v^2 = \frac{(6.7\times10^{-11})(5.97\times10^{24})}{2.66 \times 10^7}$$

$v = 3.869 \times 10^{3}$ m s^{-1}

(vi) Calculate the minimum time it takes a GPS signal to travel from the satellite to a receiver on the surface of the Earth.

$$speed = \frac{distance}{time}$$

Speed = speed of radio wave which travels at the speed of light: 3×10^{8} m s^{-1}
Distance travelled = distance from satellite to surface of the Earth = 2.023×10^{7} m

$$time\ taken = \frac{distance\ travelled}{speed} = \frac{2.023 \times 10^7}{3 \times 10^8}$$

time = 0.067 seconds

(vii) Explain why GPS satellites are not classed as geostationary satellites.

Periodic time of the satellite is 12 hours, while the periodic time of the Earth is 24 hours (so the satellite won't always be over the same spot on the Earth).

(viii) What type of electromagnetic radiation has the next lowest frequency?

Microwaves

2014 Question 12 (a)

(i) State Hooke's law.

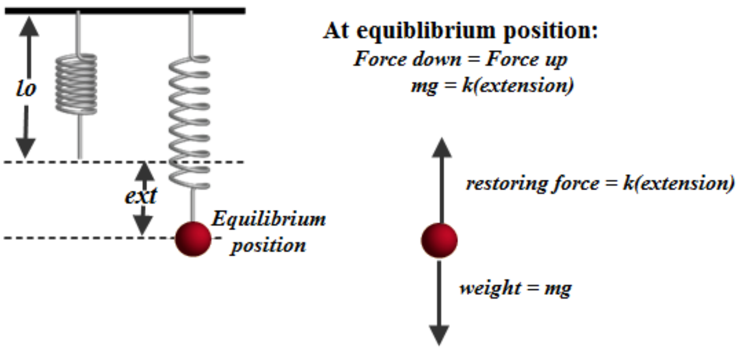
Hooke's Law states that when an object is stretched or compressed the restoring force is directly proportional to the displacement, provided the elastic limit is not exceeded.

(ii) What is the new length of the spring?

k = 12 N m^{-1}

l_0 = 25 mm = 0.025 m

m = 0.02 kg



After the mass has been attached it will come to rest at a new equilibrium position (E.P.) where

force down = force up

force down = mg = 0.02g

force up = k(extension)

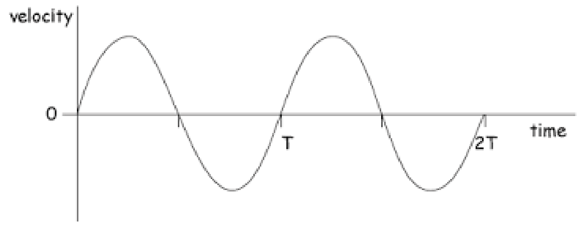
force down = force up

0.02g = 12(ext)

ext = 0.0163 m

New length = 0.025 + 0.0163 = 0.0413 m

(iii) Sketch a velocity-time graph of the motion of the object.



(iv) Calculate the period of oscillation of the object.

{Note that the 5 mm here is not relevant}

$T = \frac{2\pi}{\omega}$, so first we need to calculate ω.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{0.02}}$$

$$\omega = 24.5$$

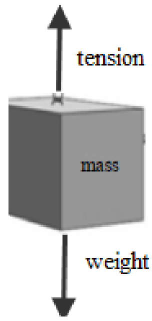
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{24.5}$$

T = 0.256 s

2013 Question 11

(a) **Seismic waves can be longitudinal or transverse. What is the main difference between them?**
A longitudinal wave is a wave where the direction of vibration is *parallel* to the direction in which the wave travels.
A transverse wave is a wave where the direction of vibration is *perpendicular* to the direction in which the wave travels.

(b) **How far is the station from the centre of the earthquake?**
speed = distance/time distance = (speed)(time) = (5000)(27) = 135000 m



(c) **Draw a diagram to show the forces acting on the suspended mass when the seismometer is at rest.**
Weight acting downwards and tension acting upwards

(d) **What is the value, in kilograms, of the suspended mass?**
Weight = mg
49 = m(9.8)
m = 5 kg

(e) **What type of motion does the frame have when it moves relative to the mass?**
Simple harmonic motion

(f) **Give an equation for the acceleration of the ground in terms of the periodic time of the wave motion and the displacement of the ground.**
The general equation for shm is $a = \omega^2 s$

We need to introduce an expression for periodic time (T) into this somehow.
We know that the relationship between T and ω is: $T = \frac{2\pi}{\omega}$ so $\omega = \frac{2\pi}{T}$

$\omega^2 = \frac{4\pi^2}{T^2}$

We can now substitute this expression for ω^2 into $a = \omega^2 s$

$$a = \frac{4\pi^2}{T^2} s$$

(g) **If the period of the ground motion was recorded as 17 seconds and its amplitude was recorded as 0.8 cm, calculate the maximum ground acceleration at the recording station.**
In the equation for simple harmonic motion $a = \omega^2 s$, s represents displacement.

Amplitude represents maximum displacement, so 0.8 cm (0.0008 m) represents s_{max}

$a = \omega^2 s$ $a_{max} = \omega^2 s_{max}$

$$a_{max} = \frac{4\pi^2}{17^2}(0.008)$$

$a_{max} = 0.0011 \text{ m s}^{-2}$

(h) **Explain why an emf is generated in the coil.**
The magnetic flux passing through the coil is changing

2013 Question 6

(i) **State Newton's law of universal gravitation**.
Newton's Law of Gravitation states that any two point masses in the universe attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

(ii) **Explain what is meant by angular velocity.**
Angular Velocity is the rate of change of angle with respect to time.

(iii) **Derive an equation for the angular velocity of an object in terms of its linear velocity when the object moves in a circle.**

The definition of an angle (in radians) is: $\boldsymbol{\theta = \frac{s}{r}}$

Now divide both sides by *t* to get: $\boldsymbol{\frac{\theta}{t} = \frac{s}{tr}}$

But $\omega = \frac{\theta}{t}$ and $v = \frac{s}{t}$ so we get `$\boldsymbol{\omega = \frac{v}{r}}$

(iv) **Calculate (*a*) the angular velocity, (*b*) the linear velocity, of the ISS.**
It takes the ISS 92 minutes 50 seconds to do one complete orbit.
This corresponds to the periodic time T, but we need to convert it into seconds.
92 minutes = 5520 seconds
T = 92 minutes 50 seconds = 5570 seconds
$\omega = \frac{2\pi}{T} = \frac{2\pi}{5570}$ = 1.1×10^{-3} ***rad s⁻¹***

$v = r\omega$
r in this context corresponds to the distance between the ISS and the centre of the Earth
r = radius of the Earth *plus* height of the ISS above the surface of the Earth.
$r = (6.37 \times 10^6) + (4.13 \times 10^5) = 6.783 \times 10^6$

$v = r\omega$
$v = (6.783 \times 10^6)(1.1 \times 10^{-3}) = 7651.5$ m s^{-1}

(v) **Name the type of acceleration that the ISS experiences as it travels in a circular orbit around the earth.**
Centripetal
What force provides this acceleration?
Gravitational force (note that you can't use the word 'gravity')

(vi) **Calculate the attractive force between the earth and the ISS.**
{We don't have enough information to use Newton's gravitational law formula, but we can use the fact the ISS is travelling in a circular orbit, and so use the equation for centripetal force; $F_c = \frac{mv^2}{r}$

m in this context corresponds to the mass of the orbiting body, and we were told in the question that the mass of the ISS is is 4.5×10^5 *kg}*

$F_c = \frac{(4.5\times10^5)(7651.5)^2}{6.783 \times 10^6}$ $F = 3.884 \times 10^6$ N

(vii) **Hence or otherwise, calculate the mass of the earth**.

Centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{GM_1M_2}{d^2}$$

Now cancel one *m* and one *d* on both sides (both *d* and *r* represent the same distance in this context)

$v^2 = \frac{GM}{r}$ $\quad M = \frac{rv^2}{G}$ $\quad M = \frac{(6.783 \times 10^6)(7651.5)^2}{6.7\times10^{-11}}$ $\quad M = 5.95 \times 10^{24}$ kg

(other methods also acceptable – can you think of two others?)

(viii) **Why do occupants of the ISS experience apparent weightlessness?**

They are in freefall // ISS accelerating at the same rate as occupants

(ix) **What is the period of a geostationary communications satellite?**

One day

2012 Question 12 (a)

(i) What is the angular velocity of the hammer during its final swing?

$T = \frac{2\pi}{\omega}$ $\qquad$ $\omega = \frac{2\pi}{0.8}$ $\qquad$ $\omega = 7.85 \text{ rad s}^{-1}$

(ii) Even though the hammer moves at a constant speed, it accelerates. Explain.

Acceleration corresponds to a change in velocity, and velocity has both a magnitude *and* direction, so if *either* of these components changes then the velocity changes and the object accelerates.
So in this case the direction changes continuously so the hammer is accelerating.

(iii) Calculate the acceleration of the hammer during its final swing.

$a = \omega^2 r$ $\qquad$ $a = (7.85)^2(2)$ $\qquad$ $a = 123.37 \text{ m s}^{-2}$ *towards the centre of orbit*

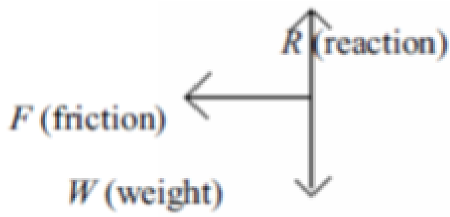
(iv) Calculate the kinetic energy of the hammer as it is released.

K.E. = $\frac{1}{2} mv^2$ $\qquad$ K.E. = $\frac{1}{2} m(r\,\omega)^2$ $\qquad$ $= \frac{1}{2}(7.26)[(2)(7.85)]^2$ $\qquad$ K.E. = 896 J

2011 Question 6 (*c*)

(i) Draw a diagram showing the forces acting on the child as the merry-go-round rotates.

See diagram



(ii) What is the maximum angular velocity of the merry-go-round so that the child will not fall from it, as it rotates?

$F = m\omega^2 r$

$50 = 30\,\omega^2(2.2)$

$\omega = 0.842 \text{ rad s}^{-1}$

(iii) If there was no force of friction between the child and the merry-go-round, in what direction would the child move as the merry-go-round starts to rotate?

The child would remain stationary

{The question wasn't phrased very well as it suggests that the child was going to move}

2011 Question 12 (*a*)

(i) **State Hooke's law.**

For a stretched string the restoring force is proportional to displacement

(ii) **Why does the body vibrate with simple harmonic motion?**

The acceleration is proportional to the displacement

(iii) **Calculate the frequency of vibration of the body**

The general expression for simple harmonic motion is $a = -\omega^2 s$.

The equation in this question is $\boldsymbol{a = -16\,s}$

Therefore we can assume that $\omega^2 = 16$ therefore $\omega = 4$

$T = \frac{2\pi}{\omega}$ and $f = \frac{1}{T} = \frac{\omega}{2\pi}$ Therefore $f = \frac{\omega}{2\pi} = \frac{4}{2\pi} = 0.64$ Hz

Note that we can ignore the negative sign in the equation $a = -\omega^2 s$. It is there to reflect that the acceleration and displacement are opposite in direction.

(iv) **What is the magnitude of (i) the maximum force, (ii) the minimum force, which causes the body's motion?**

$a = \omega^2 s$.

F_{max} occurs when *acceleration* is a maximum which according to our equation occurs when *displacement (s)* is a maximum.

Maximum displacement = amplitude = 5 cm = 0.05 m

$a_{max} = (16)(0.05) = 0.80$

$F_{max} = (m)(a_{max}) = (0.250)(0.80) = 0.20$ N

The *minimum* force occurs when displacement is 0, so acceleration is 0, so $F_{min} = 0$

2010 Question 6

(i) State Newton's law of universal gravitation.

The force between any two point masses is proportional to product of masses and inversely proportional to square of the distance between them.

(ii) Use this law to calculate the acceleration due to gravity at a height above the surface of the earth, which is twice the radius of the earth.

Here we will use the relationship $g = \frac{GM}{d^2}$

This looks like we need to know the mass of the Earth to calculate *g*, but we can actually do this without knowing the mass of the Earth.

Note that 2d above surface is 3d from earth's centre

g is proportional to $\frac{1}{d^2}$

This means that if *d* goes up by a factor of 3 (gets 3 times bigger), *g* will go down by a factor of 9 (gets 9 times *smaller*)

$g_{new} = \frac{9.81}{9}$ $\qquad$ $g_{new} = 1.09 \text{ m s}^{-2}$

(iii) Explain why the spacecraft continues on its journey to the moon, even though the engines are turned off.

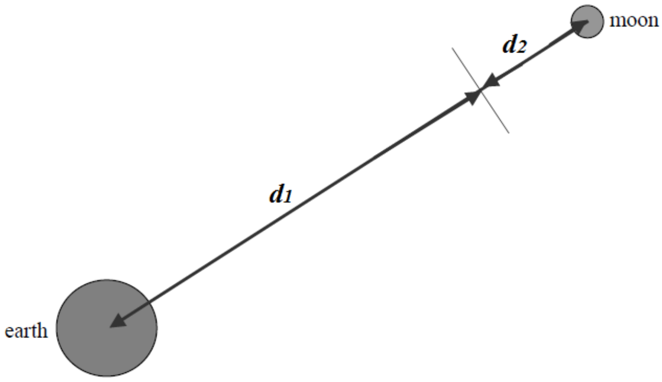
There are no external forces acting on the spacecraft so from Newton's 1st law of motion the object will maintain its velocity.

(iv) Describe the variation in the weight of the astronauts as they travel to the moon.

Weight decreases as the astronaut moves away from the earth and gains (a lesser than normal) weight as she/he approaches the moon

(v) At what height above the earth's surface will the astronauts experience weightlessness?

Gravitational pull of earth = gravitational pull of moon



d_1 = distance between astronaut and the Earth

d_2 = distance between astronaut and the Moon

m = mass of astronaut

$$\frac{Gm_E m}{d_1^{\,2}} = \frac{Gm_m m}{d_2^{\,2}}$$

Cancel G and m on both sides and rearrange to get

$$\frac{M_E}{M_M}(=81) = \frac{d_1^2}{d_2^2} \qquad 9 = \frac{d_1}{d_2}$$

$d_1 = 9d_2$

Note also that $d_1 + d_2$ = distance between the Earth and the Moon = 3.84×10^8 m

$9d_2 + d_2$ = distance between the Earth and the Moon = 3.84×10^8 m

$10\,d_2 = 3.84 \times 10^8$ $\qquad$ $d_2 = 3.84 \times 10^7$ $\qquad$ $d_1 = 3.356 \times 10^8$

Height *above the earth* = $(3.356 \times 10^8) - (6.36 \times 10^6) = 3.39 \times 10^8$ m

(vi) The moon orbits the earth every 27.3 days. What is its velocity, expressed in metres per second?

$v = \frac{2\pi r}{T}$ $\qquad$ $v = \frac{2\pi(3.84 \times 10^8)}{27.3 \times 24 \times 24 \times 60}$ $\qquad$ $v = 1022.9 \text{ m s}^{-1}$

(vii) Why is there no atmosphere on the moon?

The gravitational force is too weak to sustain an atmosphere.

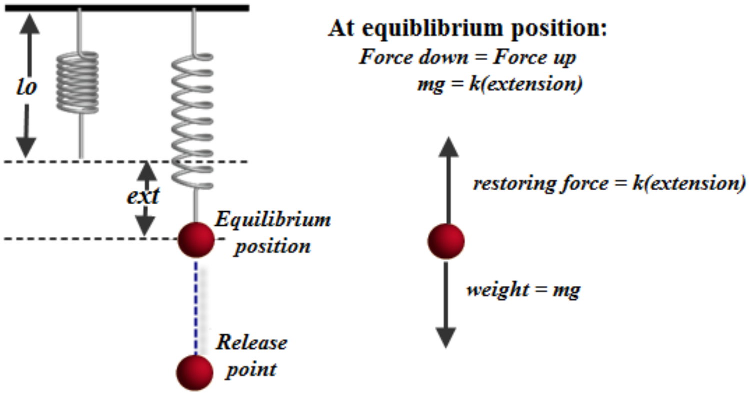
2009 Question 12 (a)

(i) State Hooke's law.

When a string is stretched the restoring force is proportional to the displacement.

(ii) Calculate the spring constant.

Natural length = l_0 = 300 mm = 0.3 m
Extension = 30 mm = 0.03 m
m = 0.5 kg



After the mass has been attached it will come to rest at a new equilibrium position where force down = force up
Force down = mg = (0.5)(g)
Force up = k(extension) = (k)(0.03)

Force down = Force up
0.5g = k(0.03)

$$k = \frac{(0.5)(9.8)}{0.03} \quad \Rightarrow k = 163.3 \text{ N m}^{-1}$$

(iii) The sphere is then pulled down until the spring's length has increased to 350 mm and is then released.
Describe the motion of the sphere when it is released.

It executes simple harmonic motion because the displacement is proportional to the acceleration.

(iv) What is the maximum acceleration of the sphere?

$a = \omega^2 x$ so acceleration will be a maximum when displacement from equilibrium position is a maximum.
Displacement is a maximum at the release point, which is a distance of 20 mm or 0.02 m from equilibrium position. So x in this context = 0.02 m

However we also need to calculate ω^2
To find ω^2 we use the relationship $\omega^2 = \frac{k}{m} = \frac{163.3}{0.5}$

$a = \omega^2 x \qquad \Rightarrow a = (\frac{163.3}{0.5})(0.02) \qquad \Rightarrow a = 6.532 \text{ m s}^{-2}$

2008 Question 6

(i) State Newton's law of universal gravitation.
Newton's Law of Gravitation states that any two point masses in the universe attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

(ii) What type of force is required to keep the ISS in orbit?
Gravity

(iii) What is the direction of this force?
Towards the centre of the orbit / inwards / towards the earth

(iv) Calculate the acceleration due to gravity at a point 400 km above the surface of the earth.
{note that 'd' is the distance from a point 400,000 m above the earth to the ***centre*** *of the earth}*

$\frac{Gm_1m_2}{d^2} = mg \Rightarrow g = \frac{GM}{d^2} \quad \Rightarrow g = \frac{(6.6 \times 10^{-7})(6.0 \times 10^{24})}{(400\,000 + 6.4 \times 10^6)^2} \qquad \Rightarrow g = 8.6 \text{ m s}^{-2}$

(v) An astronaut in the ISS appears weightless. Explain why.
He is in a state of free-fall (the force of gravity cannot be felt).

(vi) Derive the relationship between the period of the ISS, the radius of its orbit and the mass of the earth.
Centripetal force = gravitational force

$\frac{mv^2}{r} = \frac{Gm_1m_2}{d^2} \qquad \boxed{\frac{GM}{R} = v^2}$

$\Rightarrow v = \frac{2\pi R}{T} \qquad \Rightarrow v^2 = \frac{4\pi^2R^2}{T^2}$

Equating both equations for v^2 we get $T^2 = \frac{4\pi^2R^3}{GM}$

(vii) Calculate the period of an orbit of the ISS.

$T^2 = \frac{4\pi^2R^3}{GM} \qquad T^2 = \frac{4\pi^2(6.8\times10^6)^3}{(6.6\times10^{-11})(6.0\times10^{24})} \qquad \Rightarrow T^2 = 3.1347 \times 10^7 \qquad \Rightarrow T = 5.6 \times 10^3 \text{ s}$

(viii) After an orbit, the ISS will be above a different point on the earth's surface. Explain why.
The ISS has a different period to that of the earth's rotation (it is not in geostationary orbit).

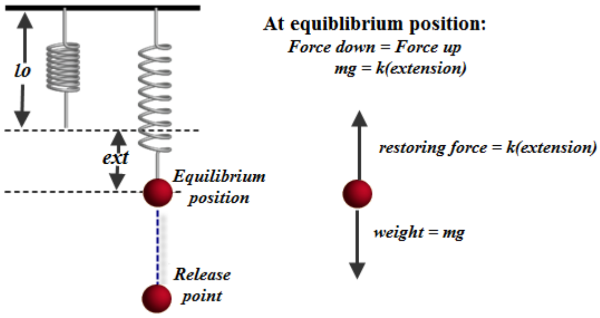
(ix) How many times does an astronaut on the ISS see the sun rise in a 24 hour period?
(24 ÷ 1.56 + 1) = 16 (sunrises).
{Why "+1"? Because the planet itself will also have turned once during this period, so even if the ISS was never moving it would have experienced this sunrise. This needs to be added to all the other 'artificial' sunrises.
Dang, that was tricky}

2007 Question 6

(i) State Hooke's law.

For a stretched string the restoring force is proportional to the extension.

(ii) Calculate its spring constant.



At equilibrium position: force down= force up

$$mg = k(\text{extension})$$

$$(0.30)(9.8) = (k)(0.085)$$

$k = 34.6 \text{ N m}^{-1}$

(iii) Derive the relationship between the acceleration of the sphere and its displacement from the fixed point.

$F = -ks \quad \Rightarrow \quad ma = -ks \quad \Rightarrow \quad a = -\frac{k}{m}s \quad \Rightarrow$

$a \propto -s \quad \Rightarrow \quad a = -k\,s$

(iv) Why does the sphere oscillate with simple harmonic motion?

We can see from the mathematical relationship above that the acceleration is proportional to displacement (and they are acting in opposite directions).

This equation is consistent with the general equation for SHM.

(v) Calculate the period of oscillation of the sphere.

From above: $\omega^2 = \frac{k}{m} \quad \Rightarrow \quad \omega^2 = \frac{34.6}{0.3} \quad \Rightarrow \omega = 10.7$

$T = \frac{2\pi}{\omega} \quad = \frac{2\pi}{10.7} \quad = 0.58 \approx 0.6 \quad \Rightarrow T = 0.6 \text{ s}$

(vi) Calculate the maximum acceleration of the sphere.

$a = -\omega^2 s$ so acceleration is proportional to displacement. So acceleration will be a maximum when displacement is a maximum.

Maximum displacement (s) is the same as the amplitude.

In this context the amplitude is the distance between release point and equilibrium position.

$$= 0.310 - 0.285 = 0.025 \text{ m.}$$

$a = -\omega^2 s \quad \Rightarrow \quad a_{max} = -(10.7)^2(0.025) \quad \Rightarrow \quad a_{max} = (-)\ 2.89 \text{ m s}^{-2}$

(vii) Calculate the length of the spring when the acceleration of the sphere is zero.

This occurs at equilibrium position $\Rightarrow l = 0.285$ m

2006 Question 6

(i) Define velocity.

Velocity is the rate of change of displacement with respect to time.

(ii) Define angular velocity.

Angular velocity is the rate of change of angle with respect to time.

(iii) Derive the relationship between the velocity of a particle travelling in uniform circular motion and its angular velocity.

$\theta_{(in\ radians)} = \frac{arc\ length}{radius}$ $\quad \theta = \frac{s}{r}$

$\frac{\theta}{t} = \frac{s}{tr}$ {divide both sides by t}

$\frac{\theta}{t} = \frac{s}{t} \times \frac{1}{r}$ but $\frac{\theta}{t} = \omega$ and $\frac{s}{t} = v$

$\omega = v \times \frac{1}{r}$ $\Rightarrow$ $v = r\omega$

(iv) What is the velocity of the ball?

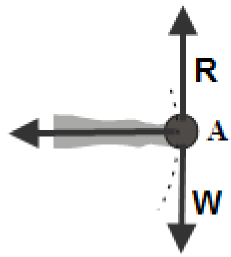
$v = r\omega = (0.70)(10) = 7.0\ \text{m s}^{-1}$

(v) How long does the ball take to complete one revolution?

$velocity = \frac{distance}{time}$ $\quad time = \frac{distance}{velocity}$

{the distance corresponds to the circumference of the circle = 2πr}

$time = \frac{2\pi(0.70)}{7}$ = 0.63 s



(vi) Draw a diagram to show the forces acting on the ball when it is at position A.

Weight (W) downwards; reaction (R) upwards; force to left (due to friction or curled fingers)

(vii) Calculate the maximum height, above the ground, the ball will reach.

$v^2 = u^2 + 2as$ $\Rightarrow$ $0 = (7)^2 + 2(-9.8)s$ $\Rightarrow s = 2.50$ m max. height = 2.5 + 1.30 = 3.8 m

(viii) Calculate the time taken for the ball to hit the ground after its release from A.

Time to go from point A to max. height: {Use $v = u + at$} $0 = 7-(9.8)t$ $t = 0.71$ s

Time to go from max. height to ground: {Use $s = ut + \frac{1}{2}at^2$ } $3.8 = 0(t) + 4.9t^2$ $t = 0.88$ s

Total time = 0.71 + 0.88 = 1.59 s

Alternative method:

{When it hits the ground it has ended up 1.3 m below where it started, so s = -1.3m}

$s = ut + \frac{1}{2}at^2$ $\quad -1.30 = 7t - \frac{1}{2}(9.8)t^2$ $\quad 4.9t^2 - 7t - 1.3 = 0$

Use $t = \frac{-b \mp \sqrt{b^2-4ac}}{2a}$ where a=4.9, b=-7, c=-1.3 $\quad$ t = 1.59 s

2005 Question 6

(i) Define angular velocity.
Angular velocity is the rate of change of displacement with respect to time.

(ii) Define centripetal force.
The force - acting in towards the centre - required to keep an object moving in a circle is called centripetal force.

(iii) State Newton's Universal Law of Gravitation.
Newton's Law of Gravitation states that any two point masses in the universe attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

(iv) Derive the relationship between the period of the satellite, the mass of Saturn and the radius of the orbit.
Centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{Gm_1m_2}{d^2} \qquad \boxed{\frac{GM}{R} = v^2}$$

$$\Rightarrow \quad v = \frac{2\pi R}{T} \qquad \Rightarrow \quad v^2 = \frac{4\pi^2 R^2}{T^2}$$

Equating both equations for v^2 we get $T^2 = \frac{4\pi^2 R^3}{GM}$

(v) The period of the satellite is 380 hours. Calculate the radius of the satellite's orbit around Saturn.

$$T^2 = \frac{4\pi^2 R^3}{GM} \qquad R^3 = \frac{GMT^2}{4\pi^2}$$

T = (380)(60)(60) = 1.37 × 10^6 seconds

$$R^3 = \frac{(6.7\times10^{-11})(6.0\times10^{24})(1.37\times10^{6})^2}{4\pi^2}$$

R = 1.2 × 10^9 m

(vi) How long does it take the signal to travel to earth?
{note that radio waves travel at the speed of light}

$$speed = \frac{distance}{time} \qquad time = \frac{distance}{speed} \qquad time = \frac{1.2\times10^{12}}{3.0\times10^{8}}$$

t = 4000 s

(vii) It is noticed that the frequency of the received radio signal changes as the satellite orbits Saturn. Explain why.
As the satellite orbits Saturn it is at times moving away from us and at other times moving towards us while all the time emitting radio waves. So from the Doppler effect the frequency will seem to change also.

2004 Question 12 (a)

(i) State Newton's universal law of gravitation.
Newton's law of gravitation states that any two point masses in the universe attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

(ii) What provides this centripetal force?
Gravitational pull of the sun.

(iii) In what direction does this centripetal force act?
Towards the centre.

(iv) Give an expression for centripetal force.

$$F_c = \frac{mv^2}{r}$$

(v) Calculate the mass of the sun.

$$F_g = \frac{Gm_1m_2}{d^2} \quad \text{and} \quad F_c = \frac{mv^2}{r} \qquad \Rightarrow \frac{GM}{R} = v^2 \quad \Rightarrow M_s = \frac{RV^2}{G}$$

$$M_s = \frac{(1.5 \times 10^{11})(3.0 \times 10^4)^2}{6.7 \times 10^{-11}} \qquad M_s = 2.0 \times 10^{30} \text{ kg.}$$

2002 Question 6

(i) State Newton's second law of motion.
Newton's second law of motion states that *the rate of change* of an object's momentum is directly proportional to the force which caused it, and takes place in the direction of the force.

(ii) Name this law and give a statement of it.
Hooke's law states that when an object is stretched (or compressed) the restoring force is directly proportional to the displacement, provided the elastic limit is not exceeded.

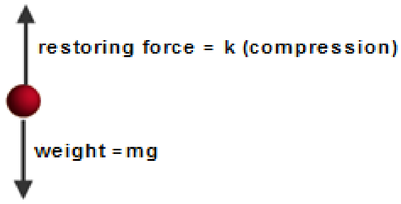
(iii) Give the name for this type of motion and describe the motion.
Simple harmonic motion; an object is said to be moving with simple harmonic motion if its acceleration is directly proportional to its distance *from* a fixed point in its path.

(iv) Give two other examples of systems that obey this law.
Stretched elastic, pendulum, oscillating magnet, springs of car, vibrating tuning fork, object bobbing in water waves.

(v) Calculate the value of k, the constant for the springs of the bike.
Force up = restoring force (which is proportional to *compression*): $F_{up} = -k(\text{compression})$
Force down = weight = mg $F_{down} = mg$
At equilibrium position, force down = force up

$$mg = -k(s) \quad \Rightarrow (60)(9.8) = k(.005) \Rightarrow$$

$$k = 1.2 \times 10^5 \text{ N m}^{-1}$$



(vi) Calculate the period of oscillation of the cyclist.

$$\omega^2 = \frac{k}{m} \qquad \omega^2 = \frac{1.2\times10^5}{80} \qquad \omega = 38 \text{ s}^{-1} \qquad T = \frac{2\pi}{\omega} \qquad T = \frac{2\pi}{38} \qquad T = 0.16 \text{ s}$$

(vii) Calculate the number of oscillations of the cyclist per second.

$$f = \frac{1}{T} \qquad f = \frac{1}{0.16} \qquad f = 6.25 \text{ Hz}$$

Number of full oscillations is 6